AXISYMMETRICAL CREEP PROBLEMS OF CIRCULAR CYLINDRICAL SHELLS

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PMM Vol.28, № 6, 1964, pp.1040-1047

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(Received August 14, 1964)

Approximate equations have been obtained in the paper [1], which describe elastic-plastic deformations of certain classes of shells. As a basic of the analysis there was taken the double-layered model, which replaces the real shell. Further simplifications followed in some cases from the particular properties of the state of stress and deformation. For the axisymmetrical deformation of a cylindrical shell the equations turned out to be exact within the bounds of the fundamental assumptions of the theory of shells and applied to the chosen model. In the paper mentioned above it was noted that the same equations are valid to describe the steady creep flow of shells. Hence, the introduction of the double-layered model must be looked upon as one of the approximate methods of the true dependence of the velocities of deformation and the rates of change of curvature from the stresses and moments. As it is known, it is practically impossible to obtain these dependences in an explicit form. Here it is proposed a further development of the theory, exposed in [1], applied to the problem of creep of a circular cylindrical shell, fixed and loaded in an axisymmetrical way. The creep equations of the shell are deduced taking into consideration the axial force , the variational principle is formulated, which allows to construct the approximate solution in an effective way.

1. Let us consider a circular cylindrical shell with the radius of the middle surface equal to a . Let us take the x-axis along the generatrix of the cylinder, all the quantities attributed to this direction, will be designated by the subscript 1, and the quantities, attributed to the perpendicular direction will be designated by the subscript 2. The velocities of deformation of the middle surface will be ϵ_1 and ϵ_2, the rate of change of curvature of the generatrix will be designated by $\varkappa_1$, the rate of change of curvature in the circumferential direction will be $\varkappa_2 = 0$. The real shell of thickness $2H$ is replaced by a model shell, which consists of two layers, each of thickness δ , the distance between the middle lines of the layers is $2h$. We will assume, that the velocities of deformation do not change along the thickness of each layer, therefore, the stresses are distributed uniformly. The quantities attributed to the external layer, will be denoted by the (+) sign, and the quantities attributed to the inner layer

will be denoted by the (–) sign. On account of the hypothesis of straight normals, the velocities of deformation of the layers are expressed in the following way:

$$\varepsilon_1{}^+ = \varepsilon_1 + \varkappa_1 h, \qquad \varepsilon_1{}^- = \varepsilon_1 - \varkappa_1 h, \qquad \varepsilon_2{}^+ = \varepsilon_2{}^- = \varepsilon_2$$

For a steady creep flow the stresses are determined in therms of the deformations by Formulas

$$\sigma_1{}^+ = \frac{4}{3}\frac{\sigma_0{}^+}{\varepsilon_0{}^+}\left[\varepsilon_1 + \varkappa_1 h + \frac{1}{2}\varepsilon_2\right], \qquad \sigma_1{}^- = \frac{4}{3}\frac{\sigma_0{}^-}{\varepsilon_0{}^-}\left[\varepsilon_1 - \varkappa_1 h + \frac{1}{2}\varepsilon_2\right]$$

$$\sigma_2{}^+ = \frac{4}{3}\frac{\sigma_0{}^+}{\varepsilon_0{}^+}\left[\varepsilon_2 + \frac{1}{2}\varepsilon_1 + \frac{1}{2}\varkappa_1 h\right], \qquad \sigma_2{}^- = \frac{4}{3}\frac{\sigma_0{}^-}{\varepsilon_0{}^-}\left[\varepsilon_2 + \frac{1}{2}\varepsilon_1 - \frac{1}{2}\varkappa_1 h\right] \qquad (1.1)$$

Here σ_0 and ε_0 correspond to the intensities of the stresses and to the velocities of deformation, the relation between them is established by the creep law, found from the creep experiment of a sample subjected to tension

$$\varepsilon_0 = \varepsilon_* v\,(\sigma_0 / \sigma_*)$$

In this formula ε_* and σ_* are constants, which have the dimension of the velocity of deformation and of the stress, correspondingly. Let us designate by M_1, M_2 the bending moments, and by T_1 and T_2 the atresses. It is obvious,that

$$M_1 = \delta h\,(\sigma_1{}^+ - \sigma_1{}^-), \qquad T_1 = \delta\,(\sigma_1{}^+ + \sigma_1{}^-), \qquad T_2 = \delta\,(\sigma_2{}^+ + \sigma_2{}^-) \qquad (1.2)$$

From (1.1) and the first two relations (1.2) it follows that

$$\varepsilon_1 \pm \varkappa_1 h = \frac{3}{8\delta h}\frac{\varepsilon_0{}^\pm}{\sigma_0{}^\pm}(M_1 \pm hT_1) - \frac{1}{2}\varepsilon_2 \qquad (1.3)$$

We will introduce the following nondimensional parameters

$$\frac{\sqrt{3}}{4}\frac{M_1}{\delta h\sigma_*} = m, \qquad \frac{\sqrt{3}}{4}\frac{T_1}{\delta\sigma_*} = \tau, \qquad \frac{\varepsilon_2}{\varepsilon_*} = u, \qquad \frac{\varepsilon_0\sigma_*}{\sigma_0\varepsilon_*} = \omega \qquad (1.4)$$

(in parer [1] by ω the inverse value was designated).

Now, the relation (1.3) can be rewritten in the following way:

$$(\varepsilon_1 \pm \varkappa_1 h)/\varepsilon_* = {}^1/_2\sqrt{3}\,(m \pm \tau)\,\omega^\pm - {}^1/_2 u \qquad (1.5)$$

For the intensities of the velocities of deformation in the layers we have

$$(\varepsilon_0{}^\pm)^2 = {}^4/_3\,[\varepsilon_2{}^2 + (\varepsilon_1 \pm \varkappa_1 h)^2 + \varepsilon_2\,(\varepsilon_1 \pm \varkappa_1 h)]$$

Taking into consideration (1.5) and (1.4) we obtain

$$(v^\pm)^2 = u^2 + (m \pm \tau)^2\,(\omega^\pm)^2 \qquad (1.6)$$

By definition, ω can be considered as a function of $v = \varepsilon_0/\varepsilon_*$, therefore, v is also a known function of ω, in this way, Equation (1.6) determines ω^+ and ω^-, which depend on the values u, m and τ. With the help of (1.1), (1.2) and (1.5) we obtain for the stress T_2 and for the rate of change of $\varkappa_1$ the following expressions: (1.7)

$$T_2 = \delta\sigma_*\left[u\left(\frac{1}{\omega^+} + \frac{1}{\omega^-}\right) + \frac{2}{\sqrt{3}}\tau\right], \qquad \varkappa_1 = \frac{\sqrt{3}}{4}\frac{\varepsilon_*}{h}\,[(m+\tau)\,\omega^+ + (m-\tau)\,\omega^-]$$

Formulas (1.6) and (1.7) express all parameters, which appear in the equations of a cylindrical shell, in terms of m , u and τ . The parameters δ and h of the model shell are chosen depending on the thickness of the real shell and on the creep law. Let us require that the behavior of the real shell and the double-layered model be the same in the membrane state and in the pure bending state. From these conditions it follows that

$$\delta = H, \quad \delta hs(\varkappa h) = \int_0^H s(\varkappa z)\, zdz \tag{1.8}$$

Here s is a function, which gives the dependence of the stress from the velocity of the creep flow during tension $\sigma = \sigma_* s(\varepsilon / \varepsilon_*)$.

The determination of the value h , generally depends on $\varkappa$, only for a creep law given in a power form. From (1.8) it follows that

$$h = \left(\frac{n}{1+2n}\right)^{n/(n+1)} H$$

In this way, the application of the double-layered model is mostly justified for a power law.

It ought to be noted, that all the authors who considered the creep of shells, have been forced to use some kind of approximate dependence of the velocity of deformation and the rate of change of curvature on the stresses and moments [2 to 4], because it has not been possible to write the exact relations. The introduction of a double-layered model by the described method is one of the ways of approximation of the relations mentioned above.

2. The equilibrium equations for a symmetrically loaded circular cylindrical shell will be the following:

$$\frac{dT_1}{dx} = 0, \qquad \frac{dQ}{dx} + \frac{T_2}{a} + q = 0, \qquad \frac{dM_1}{dx} + Q = 0 \tag{2.1}$$

Here q is the normal load and Q is the shearing force. From the second and third equations of (2.1) it follows that

$$d^2M_1 / dx^2 - a^{-1}T_2 - q = 0 \tag{2.2}$$

The deformation in the circumferential direction and the rate of change of curvature of the generatrix are connected by the following equation of compatibility:

$$d^2\varepsilon_2 / dx^2 + a^{-1}\varkappa_1 = 0 \tag{2.3}$$

Let us introduce a nondimensional parameter

$$\xi = x / b, \qquad b = (^{16}/_3)^{1/4}\sqrt{ah}$$

and by using (1.4) and (1.7) we will pass to the new variables m and u in Equations (2.2) and (2.3). As a result we obtain a system of equations

$$m'' - u\left(\frac{1}{\omega^+} + \frac{1}{\omega^-}\right) - \frac{2}{\sqrt{3}}\tau + 2p = 0 \qquad \left(p = -\frac{qa}{2\delta\sigma^{\alpha}{}_*}\right) \tag{2.4}$$

$$u'' + (m+\tau)\,\omega^+ + (m-\tau)\,\omega^- = 0 \tag{2.5}$$

As a consequence of the first equation of (2.1), τ is here a constant, the primes indicate the differentiation with respect to ξ . The magnitudes

ω^+ and ω^- are found from Equation (1.6), which for the power creep law $\varepsilon / \varepsilon_* = (\sigma / \sigma_*)^n$ takes the form

$$(\omega\pm)^{2n/(n-1)} = u^2 + (m \pm \tau)^2 (\omega\pm)^2 \qquad (2.6)$$

Let l be the nondimensional length of the shell. Let us consider the functional

$$N = \int_0^l \Big[u'm' + \frac{1}{2}\psi(\omega^+) + \frac{1}{2}\psi(\omega^-) - (m+\tau)^2\omega^+ - (m-\tau)^2\omega^- + $$
$$+ 2\Big(\frac{\tau}{\sqrt{3}} - p\Big) u\Big] d\xi \qquad \Big(\psi(\omega) = \int \frac{dv^2}{\omega} = \frac{2}{\varepsilon_*\sigma_*}\int \sigma_0 d\varepsilon_0\Big) \qquad (2.7)$$

It is not difficult to check, that Equations (2.4) and (2.5) will be the Euler equations for this functional for the natural boundary conditions which follow from Equation

$$(u'\delta m + m'\delta u)|_0^l = 0 \qquad (2.8)$$

For the power creep law the functional (2.7) is also written in a somewhat more suitable form

$$N = \int_0^l \Big\{u'm' + \frac{n}{n+1} u^2 \Big(\frac{1}{\omega^+} + \frac{1}{\omega^-}\Big) - \frac{1}{n+1} \times$$
$$\times [(m+\tau)^2\omega^+ + (m-\tau)^2\omega^-] + 2\Big(\frac{\tau}{\sqrt{3}} - p\Big) u\Big\} d\xi \qquad (2.9)$$

3. In the absence of the axial force $\omega^+ = \omega^-$, Equations (2.4) and (2.5) are replaced by

$$m'' - \frac{2u}{\omega} + 2p = 0, \qquad u'' + 2m\omega = 0 \qquad (3.1)$$

the relation (1.6) is replaced by

$$v^2(\omega) = u^2 + m^2\omega^2 \qquad (3.2)$$

and the functional (2.7) takes the form

$$N = \int_0^l [u'm' + \psi(\omega) - 2m^2\omega - 2pu]\, d\xi \qquad (3.3)$$

During the variation of this functional the functions $u(\xi)$ and $m(\xi)$ are considered as independent. However, it can be considered that either the first or the second of the equations (3.1) is satisfied. In this case only one function is given independently, and the second one is expressed in terms of the first. With this, the functional (3.3) is converted into a functional of Lagrange type, or in a functional of Castigliano type.

Let us assume, for example, that the second equation of (3.1) is satisfied. This means that the variational function $u(\xi)$ is given, while the function $m(\xi)$ is expressed in terms of $u(\xi)$. Let us integrate by parts the first term under the integral sign in Expression (3.3). By taking into consideration the second equation of (3.1) we obtain

$$N = 2\int_0^l (^1/_2\psi - pu)\, d\xi + mu'|_0^l \qquad (3.4)$$

According to (2.8) the function $\frac{1}{2}\psi(\omega)$ represents a nondimensional creep potential, which represents the analog of an elastic potential in the corresponding problem of nonlinear elasticity. The integral of $pud\xi$ gives the wrk of the external forces, hence, the functional N is transformed into a functional of Lagrange type. Taking into account (2.9) the variational equation will be the following:

$$2\delta\int_0^l ({}^1/_2\psi - pu)\,d\xi = (m'\delta u - m\delta u')|_0^l \tag{3.5}$$

The right-hand side represents the work of the external forces, applied to the edges of the shell. The function ψ is determined in terms of ω, but as a consequence of (3.2) the quantity ω is a function of

$$\dot{u}^2 + m^2\omega^2 = \dot{u}^2 + \tfrac{1}{4}u''^2.$$

Let us now assume, that the first equation of (3.1) is satisfied. Following exactly the same method, we will transform the functional (3.3) to the form

$$N = \int_0^l \left(\psi - \frac{2v^2}{\omega}\right) d\xi + um'|_0^l$$

The expression under the integral sign

$$\psi - \frac{2v^2}{\omega} = \frac{2}{\varepsilon_*\sigma_*}\left[\int \sigma_0 d\varepsilon_0 - \sigma_0\varepsilon_0\right] = -\frac{2}{\varepsilon_*\sigma_*}\int \varepsilon_0 d\sigma_0$$

will be the analog of the additional work for thw nonlinear elastic body. Therefore, the functional N is transformed to the Castigliano functional

$$N = -2\Phi + um'|_0^l$$

The variational equation with the consideration of (2.9) will be the following:

$$2\delta\Phi = (u\delta m' - u'\delta m)_0{}^l \tag{3.6}$$

The argument of the function $\psi - 2v^2/\omega$ will be now the expression

$$m^2 + {}^1/_4\,(m'' + 2p)_2$$

Exactly analogous results are also obtained, in the case when the axial force is different from zero, only in this case it is not possible to write an explicit expression for the Lagrange functional in terms of $u(\xi)$ and for the Castigliano functional in terms of $m(\xi)$.

4. The application of the formulated variational principle, which allows the independent prescription of the approximate functions for $u(\xi)$ and for $m(\xi)$, has some advantages. In any case, introducing two sought parameters, we can expect a better accuracy, then by the use of the ordinary variational principles, where in most cases only one parameter is introduced. Incidentally, let us notice that the condition $\delta N = 0$ which depends on the election of the functions of comparison can mean as well a maximum as a minimum of the functional N .

As an illustration, let us consider the problem of an edge effect of an infinitely long cylindrical shell, loaded with a uniformly distributed pressure. Let us consider at first that the axial force is absent, the case $\tau \neq 0$ is considered in exactly analogous way. We will take the power creep law, place the origin of the coordinates at the edge of the shell and we will solve the problem for two cases: (a) the edge is simply supported, $u(0) = m(0) = 0$, and (b) the edge is clamped, $u(0) = u'(0) = 0$.

At a sufficiently large distance from the supported edge the shell is in a membrane state, therefore, $m(\infty) = 0$, $u(\infty) = u^*$, $\omega(\infty) = \omega^*$, from the second equation of (3.1) and the relation (3.2) it follows that

$$u^* = p^n, \qquad \omega^* = p^{n-1} = u^{*\frac{n-1}{n}}$$

Let us introduce the functions $U(\xi)$ and $V(\xi)$ which satisfy the system of linear equations

$$U'' + 2\mu V = 0, \qquad V'' - 2\lambda (U - u^*) = 0 \tag{4.1}$$

and the boundary conditions

$U(0) = 0$, $V(0) = 0$ in the case (a); $U(0) = 0$, $U'(0) = 0$ in the case (b).

The solution of Equations (4.1) can be expressed in the following way:

$$U = u^* U_0(\alpha\xi), \qquad V = \beta u^{*\frac{1}{n}} V_0(\alpha\xi) \left(\alpha^4 = \lambda\mu,\ \beta = u^{*\frac{n-1}{n}} \left(\frac{\lambda}{\mu}\right)^{1/2}\right) \tag{4.2}$$

The functions U_0 and V_0 are determined by Expressions

$$U_0(x) = 1 - e^{-x}\cos x, \qquad V_0(x) = e^{-x}\sin x \quad \text{(case (a))} \tag{4.3}$$

$$U_0(x) = 1 - e^{-x}(\cos x + \sin x), \qquad V_0(x) = e^{-x}(\cos x - \sin x) \quad \text{(case (b))}$$

Let us look for the solution of the problem in consideration in the form of $u = U(\alpha\xi)$, $m = V(\alpha\xi)$, where the parameters α and β are chosen from the condition that the variation of the functional N is zero. In addition, let us assume that

$$\omega = \omega^*\omega_0 = u^{*\frac{n-1}{n}}\omega_0 \tag{4.4}$$

Introducing (4.2) and (4.4) into the first equation of (3.1), and determining in (3.2) the function v which corresponds to the power creep law, we obtain

$$\omega_0^{\frac{2n}{n-1}} = U_0^2 + \beta^2 V_0^2 \omega_0^2 \tag{4.5}$$

For the calculation of the functional (3.3) let us express it in the form (2.9) letting $\tau = 0$ and $\omega^+ = \omega^-$. For an infinite region the integral turns out to be divergent; to provide its convergence we will add some constant terms under the integral expression. For the purpose of calculation, it is convenient to choose $x = \alpha\xi$ as the variable of integration; now the primes will indicate the differentiation with respect to the variable. To within a constant multiplier we obtain for the functional N

$$N = \alpha\beta \int_0^\infty U_0' V_0' dx + \frac{1}{\alpha}\int_0^\infty \left[\frac{2n}{n+1}\left(\frac{U_0^2}{\omega_0} - 1\right) - \frac{2}{n+1}\beta^2 V_0^2\omega_0 - 2(U_0 - 1)\right] dx \tag{4.6}$$

It is evident, that with the assumed election of the functions of comparison the condition (2.9) will be satisfied in the case (a), as well as in the case (b).

Let us write Expression (4.6) in the following way:

$$N = \alpha\beta A + \frac{1}{\alpha} B(\beta)$$

The stationary conditions will be

$$\beta A - \frac{1}{\alpha^2} B(\beta) = 0, \qquad \alpha A + \frac{1}{\alpha}\frac{\partial B}{\partial \beta} = 0$$

From here it follows that

$$\frac{\partial}{\partial\beta}(\beta B) = 0, \qquad \alpha^2 = \frac{B}{A\beta} \tag{4.7}$$

For $n = 1$ we have $\beta = 1$, hence we must expect, that for $n > 1$ we will have $\beta < 1$.

Let us now pass to the construction of the approximate solutions for the boundary conditions.

Case (a). The quantity V_0 results to be considerably smaller, than U_0, the quantity ω_0 must be smaller than unity; $\beta < 1$, therefore, the first term will be the predominant one on the right-hand side of Equation (4.5). Let us assume

$$\omega_0 = U^{\frac{n-1}{n}}(1+\rho) \qquad (\rho \ll 1)$$

Substituting this expression into (4.5) and retaining only the first power of ρ, we obtain

$$\rho \approx \frac{n-1}{2n}\beta^2 V_0^2 U_0^{-2/n}$$

With the same degree of accuracy we obtain

$$B(\beta) \approx 2J_1 - \beta^2 J_2$$

Whereas $A = -\frac{1}{4}$. In the expression for $B(\beta)$ we have

$$J_1 = \int_0^\infty \left[\frac{n}{n+1} U^{\frac{n+1}{n}} - U_0 + \frac{1}{1+n}\right] dx, \qquad J_2 = \int_0^\infty V_0^2 U^{\frac{n-1}{n}} dx$$

Now from (4.7) it follows that

$$\beta^2 = \frac{2J_1}{3J_2}, \qquad \alpha^2 = -4\left(\frac{2J_1}{\beta} - J_2\beta\right) \tag{4.8}$$

We give the values of β and α calculated for $n = 1,\ldots,6$.

n	1	2	3	4	5	6
$\beta =$	1.000	0.840	0.728	0.643	0.598	0.540
$\alpha =$	1.000	0.860	0.763	0.718	0.678	0.646

For bigger values of n, the approximation, based on the assumption, that the deflections and moments are the same, as for an elastic shell, can not be considered reliable. Besides, an exact formulation of the shell problem, loaded with the pressure and supported at the edge, is contradictory to a power creep law. Indeed, $u = 0$, $m = 0$ for $x = 0$, therefore, $\omega = 0$, and the solution of Equations (3.1) in the neighborhood of the origin of the coordinates does not exist.

The reason for this is easy to understand, if we observe, that for the given boundary conditions the limiting state of an ideally rigid-plastic shell is impossible. The circumstance, that for a power creep law the diagram $\sigma_0 \epsilon_0$ is tangent to the ϵ_0 axis, creates here a similar situation. The situation can be corrected, by midifying slightly the creep law in the region of very small stresses. With the use of the variational method this difficulty does not arise.

C a s e (b) . Now $A = \frac{1}{2}$, the quantities U_0 and V_0 have the same order in a certain region, and the simplifications, found out from the consideration of a simply supported shell, do not take place here. Let us express $B(\beta)$ in the form

$$B(\beta) = 2J_1 - \frac{2}{n+1}\beta^2 J_2$$

Here

$$J_1 = \int_0^\infty \left[\frac{n}{n+1}\left(\frac{U_0^2}{\omega_0} - 1\right) - U_0 + 1\right] d\xi, \qquad J_2 = \int_0^\infty V_0^2 \omega_0 d\xi$$

The integrals J_1 and J_2 are functions of β . The condition (4.7) leads to the following relations:

$$J_1 - \beta^2 \frac{n+2}{n+1} J_2 = 0, \qquad \alpha = 4\left(\frac{J_1}{\beta} - \frac{\beta}{n+1} J_2\right) \tag{4.9}$$

The calculations, done for $n = 3$, have shown, that $\beta = 1$, for which $\alpha = 0.716$. The circumstance, that the magnitude of the moment at the clamped edge for $n = 3$ is equal to the magnitude of the moment for $n = 1$, hence, it is the same as for an elastic shell, must not be surprising. In the limiting state of an ideally plastic shell for $p = 1$ the magnitude of the moment at the clamped edge is equal to $\frac{1}{2}\sqrt{3} = 0.866$, which differes by a relatively small amount from unity.

The same problem was solved in the paper [4], with the help of the variational Lagrange equation. The deflection was given in the same form as here, and the simplification consisted in that the rate of change of the curvature was considered only a function of the moment, and the velocity of circular deformation only a function of circumferential stress. For $n = 3$, the value $\alpha = 0.659$ was found. Of course, the distribution of the bending moment turned out to be completel different. In the presence of an axial force, the computation is done by an analogous method.

For this purpose let us assume

$$\tau = \nu c^{1/n}, \quad u = \gamma c U_0, \quad m = \beta c^{1/n} V_0, \quad \omega^\pm = c^{\frac{n-1}{n}} \omega_0^\pm \tag{4.10}$$

Here U_0 is a function, which satisfies the boundary conditions for the deflection and which is unity at infinity, V_0 is a function which satisfies tne boundary conditions for the moment and which vanishes at infinity. We can choose these functions in the same way as in the absence of the axial force. Then they will depend on the argument $\alpha\xi$, the quantity α , which determines the rate of damping of the edge effect, is one of the sought variational parameters, and the second of these parameters is β .

Let us require, that $\omega_0^{\pm}(\infty) = 1$, then from (2.6) it follows that

$$1 = \gamma^2 + \nu^2$$

Equation (2.4) for $\xi = \infty$ gives

$$p = c^{1/n}\left(\gamma + \frac{\nu}{\sqrt{3}}\right) \tag{4.11}$$

The quantity τ is given, consequently, all the parameters introduced are now known, i.e.

$$c^{1/n} = \frac{\tau}{\nu}, \quad \nu = \frac{1}{\Omega}, \quad \gamma = \frac{1}{\Omega}\left(\frac{p}{\tau} - \frac{1}{\sqrt{3}}\right) \quad \left(\Omega = \left[1 + \left(\frac{p}{\tau} - \frac{1}{\sqrt{3}}\right)^2\right]^{1/2}\right) \tag{4.12}$$

Equation (2.4) takes the form

$$(\omega_0^{\pm})^{\frac{2n}{n-1}} = \gamma^2 U_0^2 + (\beta V_0 \pm \nu)^2 (\omega_0^{\pm})^2 \tag{4.13}$$

The quantities α and β are found by the previous method from the conditions (4.7), in which

$$A = \gamma \int_0^\infty U_0' V_0' dx, \qquad B(\beta) = \int_0^\infty \left\{\frac{n}{n+1}\gamma^2\left[U_0^2\left(\frac{1}{\omega_0^+} + \frac{1}{\omega_0^-}\right) - 2\right] - \right.$$

$$\left. - \frac{1}{n+1}\left[(\beta V_0 + \nu)^2 \omega_0^+ + (\beta V_0 - \nu)^2 \omega_0^- - 2\nu^2\right] - 2\gamma^2 (U_0 - 1)\right\} dx \tag{4.14}$$

The computation is reduced to the calculation of $B(\beta)$ for some values of β , after that, we find by interpolation the value, for which the first of the conditions (4.8) is satisfied. Let us observe, that $|\beta| < \gamma$ for $n > 1$.

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Translated by E.Z.